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Quantization of charges and fluxes in warped Stenzel geometry

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Abstract

We examine the quantization of fluxes for the warped Stiefel cone and Stenzel geometries and their orbifolds, and distinguish the roles of three related notions of charge: Page, Maxwell, and brane. The orbifolds admit discrete torsion, and we describe the associated quantum numbers which are consistent with the geometry in its large radius and small radius limits from both the type IIA and the M-theory perspectives. The discrete torsion, measured by a Page charge, is related to the number of fractional branes. We relate the shifts in the Page charges under large gauge transformations to the Hanany-Witten brane creation effect.

1 Introduction

Recently, a holographic duality for superconformal Chern-Simons-Matter theories in 2+1 dimensions with $\mathcal{N} = 6$ and $\mathcal{N} = 8$ supersymmetry was proposed [1,2]. These field theories have $U(N)_k \times U(N+l)_{-k}$ product gauge symmetry (where the subscripts refer to the Chern-Simons level associated with each gauge group) and bifundamental matter fields. In the large N limit, the field theory has a dual gravity description in terms of M-theory as N M2-branes on the orbifold C^4/Z_k (where the orbifold acts by rotating each of the complex planes by an angle $2\pi/k$ simultaneously) and l fractional branes. The supergravity solution corresponding to this brane system is $AdS_4 \times S^7/Z_k$, and the quantum number l is encoded in the discrete torsion of the $H_3(Z) = Z_k$ homology group of S^7/Z_k.

The M-theory background can also be described in type IIA supergravity by dimensionally reducing along the Hopf-fiber of S^7/Z_k. In this description, the geometry has the form $AdS_4 \times CP^3$ and is the effective description when $1 \ll N \ll k^5$. Homologically, CP^3 is very different from S^7/Z_k, particularly in that CP^3 has no discrete torsion cycles, but it does possess integral homology. Even for this simple example, the relationship of the spectrum of charges and fluxes in the M-theory and the type IIA descriptions is subtle.

One way to gain some perspective on the physical meaning of the defining data of the gravity side of these correspondences is to realize the superconformal field theory as either the UV or IR fixed point of a holographic renormalization group flow. For example, a superconformal Chern-Simons theory can arise as the IR fixed point of an RG flow from a Chern-Simons-Yang-Mills-Matter theory [3,4]. Several related realizations have also been considered [5]. These renormalization group flows are dual to transverse geometries which differ from R^8, and many of these constructions have the interesting property that the dual geometry admits a normalizable 4-form. In M-theory, this allows one to introduce a nontrivial 4-form flux. The freedom of tuning the 4-form flux has a specific interpretation in terms of tuning the parameters of the dual field theory, and in some examples, one can explore dynamical features such as phase transitions in the low energy effective field theory from the geometry of the supergravity dual [4,5].

In this article, we investigate the duality of $\mathcal{N} = 2$ Chern-Simons quiver theories dual to $AdS_4 \times V_{5,2}/Z_k$ where $V_{5,2}$ is a homogeneous Sasaki-Einstein seven-manifold. This duality was originally considered by Martelli and Sparks in [6]. On the field theory side, it generalizes the model of ABJM by adding a chiral multiplet in the adjoint representation to each factor of the $U(N)_k \times U(N+l)_{-k}$ gauge group. The gravity dual description can be deformed in the IR, giving rise to a geometry known as the warped Stenzel metric. At the moment, little is known about the field theory interpretation of this IR deformation. In order to facilitate

in its interpretation, it is useful to enumerate the the discrete and continuous parameters associated with this system. This is related to the problem of quantizing charges and fluxes in the gravity dual.

In type IIA (and IIB) supergravity, there is a well-known subtlety in imposing charge quantization, which arises in the example studied in this paper. The $V_{5,2}/Z_k$ geometry, reduced to IIA along the $U(1)$ isometry along which the Z_k acts, is a space M_2 which has the same homology structure as CP^3 [6]; in particular there is a nontrivial 4-cycle. Now, one might want to quantize the four-form flux through this cycle, but the natural gauge-invariant four-form

$$\tilde{F}_4 = dA_3 + H_3 \wedge A_1 \tag{1.1}$$

is not closed, and therefore its integral through the 4-cycle is not conserved and cannot be quantized. A similar issue arises in the flux of $*\tilde{F}_4$ through M_2. These apparent difficulties have also appeared in earlier examples considered in [4,5] and their resolution is well understood. The four-form flux satisfies a modified Bianchi identity,

$$d\tilde{F}_4 = -H_3 \wedge F_2 \tag{1.2}$$

so to define a conserved charge we should not integrate $\tilde{F}_4$ but a modified flux which is chosen to be closed:

$$Q_4^{Page} = \frac{1}{(2\pi l_s)^3 g_s} \int (-\tilde{F}_4 + B \wedge F_2) \ . \tag{1.3}$$

This new charge, known as the Page charge, is one of the three subtle notion of charges identified by Marolf [7]. The three charges being referred here are the Maxwell charge, brane charge, and the Page charge, and they can take distinct values in gauge theories involving Chern-Simons terms as is the case for type IIA supergravity. Each of these charges respects some, but not all, of the properties commonly associated with charges in simpler contexts: gauge invariance, conservation, localization, and integer quantization. Page charge turns out to respect conservation, localization, and integer quantization, but fails to be invariant with respect to large gauge transformations which shift the period of B_2. This ambiguity is precisely what is required to interpret the Hanany-Witten brane creation effects in the brane construction of these models is intimately connected to the duality of the field theory.

In this article, we will analyze the quantization of fluxes in $AdS_4 \times V_{5,2}/Z_k$ geometry and its Stenzel deformation from the type IIA perspective, and relate the gauge ambiguity to Hanany-Witten brane creation effects along the lines of [4,5]. In [6], it was argued that the Stenzel deformation is incompatible with the presence of discrete torsion which gives rise to a non-vanishing value of l in $U(N)_k \times U(N+l)_{-k}$. On the contrary, we find that some values of l are allowed, and explain the source of this apparent discrepancy. We will also examine the compatibility of the IIA and the M-theory perspectives.

2 Stiefel, Stenzel, and the $\mathcal{N}=2$ Chern-Simons-Quiver theory

In this section, we briefly review the construction of $\mathcal{N}=2$ Chern-Simons-Quiver theory, its gravity dual, and its Stenzel deformation. We closely follow the presentation of [6].

2.1 Stiefel cone

The starting point is a non-compact Calabi-Yau 4-fold

$$z_0^n + z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0 \tag{2.1}$$

where we take $n=2$. This geometry is a cone whose base is a Sasaki-Einstein seven manifold $V_{5,2}$, also known as the Stiefel manifold. Had one taken $n=1$, the geometry of the Calabi-Yau 4-fold would have been R^8 which is formally a cone over S^7. For $n>2$, the geometry is not a cone over a Sasaki-Einstein manifold [6].

When M2 branes are placed at the tip of the cone, we obtain a warped geometry $AdS_4 \times V_{5,2}$. The base $Y_2 = V_{5,2}$ has a torsion 3-cycle $H_3(Y_2, Z) = Z_2$.

The Z_k orbifold is taken on the $U(1)_b$ isometry which rotates

$$(z_0, z_1 + z_2, z_3 + z_4, z_1 - z_2, z_3 - z_4) \tag{2.2}$$

with weights $(0,1,1,-1,-1)$. On Y_2/Z_k, this changes the torsion group from Z_2 to $H_3(Y_2/Z_k, Z) = Z_{2k}$, so

$$\frac{1}{(2\pi l_p)^3}\int_{\Sigma_3} C_3 = \frac{l-k}{2k} \tag{2.3}$$

for Σ_3 which generates $H_3(Y_2/Z_k)$. Here we have shifted l by k compared to what is written in (3.26) of [6]. Both l and k are integers so this shift is a matter of convention in describing the supergravity background.

When reduced to IIA along the $U(1)_b$ direction parametrized by γ, the Sasaki-Einstein space Y_2/Z_k decomposes into

$$ds^2(Y_2/Z_k) = ds^2(M_2) + \frac{w}{k^2}(d\gamma + kP)^2 \tag{2.4}$$

and the IIA string frame metric becomes

$$ds^2 = \sqrt{w}\frac{R^3}{k}\left(\frac{1}{4}ds^2(\mathrm{AdS}_4) + ds^2(M_2)^2\right) \tag{2.5}$$

with

$$F_2 = k g_s l_s \Omega_2, \qquad \Omega_2 = dP\ . \tag{2.6}$$

Since

$$C_3 = A_3 + B_2 \wedge d\gamma \tag{2.7}$$

with this dimensional reduction, B_2 turns out to have the period

$$\frac{1}{(2\pi l_s)^2} \int B_2 = \frac{l}{2k} - \frac{1}{2} \ . \tag{2.8}$$

2.2 Brane construction and the Hanany-Witten effect

The field theory dual is conjectured in [6] to arise from the low energy limit of a network of D3-branes, an NS5-brane and a $(1, k)$ 5-brane in type IIB on $R^{1,2} \times S^1 \times R^2 \times C^2$. The D3-branes wind along $R^{1,2} \times S^1$. The NS5 is extended along $R^{1,2}$, one of the R in R^2 and along the curve $w_1 = -iw_0^2$ where C^2 is parametrized by (w_0, w_1). The $(1, k)$ 5-brane is extended along $R^{1,2}$, a line at an angle relative to the NS5-brane in R^2, and along $w_1 = iw_0^2$ in C^2. There may also be fractional D3-branes stretching between the NS5 and the $(1, k)$ 5-brane at $(w_0, w_1) = (0, 0)$.

In a brane configuration of this type, the Hanany-Witten brane creation effect occurs when one of the 5-branes are moved around the circle S^1 keeping the other 5-brane fixed. If there were N integer and l fractional branes to start with, moving the 5-brane once around the circle will give rise to a shift

$$\begin{aligned} N &\rightarrow N + l \\ l &\rightarrow l + 2k \ . \end{aligned} \tag{2.9}$$

2.3 Stenzel Deformation

In this subsection, we will briefly review the IR deformation of the Stiefel cone. As an algebraic curve, it amounts to deforming (2.1) to

$$z_0^2 + z_1^2 + z_2^2 + z_3^2 + z_4^2 = \epsilon^2 \ . \tag{2.10}$$

The tip of the cone is blown up by an S^4 parametrized by

$$\sum_{i=0}^{4} (\mathrm{Re} z_i)^2 = \epsilon^2, \qquad \mathrm{Im} z_i = 0 \tag{2.11}$$

and the full geometry can be viewed as the cotangent bundle over S^4. This geometry is also known as the Stenzel geometry [8] and admits an explicit metric [9]. In the notation adopted in [10], the metric takes the form

$$ds^2 = c^2(dr^2 + \nu^2) + a^2 \sum_{i=1}^{3} \sigma_i^2 + b^2 \sum_{i=1}^{3} \tilde{\sigma}_i^2 \tag{2.12}$$

where

$$\begin{aligned}
a^2(r) &= 3^{-1/4}\lambda^2 \epsilon^{3/2}(2+\cosh 2r)^{1/4}\cosh r, \\
b^2(r) &= 3^{-1/4}\lambda^2 \epsilon^{3/2}(2+\cosh 2r)^{1/4}\sinh r \tanh r \\
c^2(r) &= 3^{3/4}\lambda^2 \epsilon^{3/2}(2+\cosh 2r)^{-3/4}\cosh^3 r
\end{aligned} \tag{2.13}$$

and ν, σ_i, and $\tilde{\sigma}_i$ are left-invariant one-forms of the coset $SO(5)/SO(3)$ (for which a nice explicit basis appears in [10].)

At $r = 0$, the geometry collapses to an S^4. At large r, the geometry asymptotes to a cone over $V_{5,2}$. Formally, this geometry is similar to the deformed B_8 space [11] which collapses to an S^4 near the tip, and asymptotes to cone over a squashed 7-sphere, but there are a few important differences. One is the fact the Z_k orbifold along the $U(1)_b$ of the Stenzel geometry has fixed points at antipodal points of S^4 at $r = 0$. We will comment on other differences below.

One important feature of the Stenzel geometry is that it admits a self-dual 4-form which can be written, explicitly, as

$$\begin{aligned}
G_4 &= m\Bigg\{\frac{3}{\epsilon^3\coth^4\frac{r}{2}}\left[a^3 c\nu\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3 + \frac{1}{2}b^3 c dr\wedge\tilde{\sigma}_1\wedge\tilde{\sigma}_2\wedge\tilde{\sigma}_3\right] \\
&\quad + \frac{1}{2\epsilon^3\coth^4\frac{r}{2}}\left[\frac{1}{2}a^2 bc\epsilon^{ijk} dr\wedge\sigma_i\wedge\sigma_j\wedge\tilde{\sigma}_k + ab^2 c\epsilon^{ijk}\nu\wedge\sigma_i\wedge\tilde{\sigma}_j\wedge\tilde{\sigma}_k\right]\Bigg\} \ .
\end{aligned} \tag{2.14}$$

Because the four-form is self-dual, and the background geometry is Calabi-Yau, one can turn on this flux in eleven-dimensional supergravity without breaking supersymmetry [12]. Moreover, it gives rise to a solution where the background geometry is unmodified except for the presence of a warp factor H, as in the standard warped product ansatz

$$\begin{aligned}
ds^2 &= H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}ds_8^2 \\
F_4 &= dt\wedge dx_1\wedge dx_2\wedge d\tilde{H}^{-1} + mG_4 \ .
\end{aligned} \tag{2.15}$$

The warp factor itself can be determined by solving the four-form field equation,

$$d * G = \frac{1}{2} G\wedge G \ , \tag{2.16}$$

where in general there can be additional source terms due to the presence of explicit M2-branes.

3 Quantization of fluxes in Stiefel cones and Stenzel space

Let us now consider the quantization of fluxes in the warped Stiefel cones and Stenzel geometries in order to identify the discrete parameters characterizing the background. There are two guiding principles which we follow in carrying out the quantization. One is that quantized fluxes should be invariant under deformation of Gaussian surfaces unless the discrete unit of charge crosses the surface. The other is for the quantization condition to be invariant under string dualities.

3.1 Review of Maxwell, brane, and Page charges

We begin by considering the quantization of fluxes for the Stenzel geometry in the IIA description. While the IIA description of the Stenzel geometry is singular near the core, one still expects Gauss law considerations to lead to a consistent picture far away from the core region, where the geometry looks essentially like the warped Stiefel cone.

The relevant fluxes to consider then are the flux of $\tilde{F}_4$ through the generator of $H_4(M_2, Z)$ and the flux of $*\tilde{F}_4$ through the six cycle M_2. As we mentioned earlier, however, these fluxes depend on the radius r at which we identify the base M_2 for the background in consideration.

The resolution to these apparent difficulties is the realization that one is dealing with a situation where the Maxwell, brane, and Page charge are distinct from one another, and that care is required in applying quantization conditions on the appropriate charge.

Let us recall the specific definition of three charges. In type IIA supergravity, the four form $\tilde{F}_4 = dA_3 + H_3 \wedge A_1$ is gauge invariant and well defined but is not closed and does not respect Gauss' law. One can nonetheless compute the period of $\tilde{F}_4$ on the generator of $H_4(M_2, Z)$ in the $r \to \infty$ limit. This defines the Maxwell charge. In contrast, the period of Page flux $-(\tilde{F}_4 + B_2 \wedge F_2)$ on $H_4(M_2, Z)$ is independent of r, although it is ambiguous with respect to large gauge transformation of B_2. This quantity defines the Page charge. Finally, the amount of charge carried by a brane source through its Wess-Zumino couplings defines the brane charge. Brane charge includes the contribution of lower-brane charges from the pull-back of the NSNS 2-form in the Wess-Zumino coupling. Therefore, if the background contains a non-uniform NSNS 2-form B_2, the brane charge is not conserved with respect to changes in the position of the branes. Some of these subtleties appeared originally in [13].

The triplet of charges exists for the other forms, e.g. the six form $F_6 = *\tilde{F}_4$ and are summarized in appendix B of [4]. For the flux of $F_6 = *F_4$, is is also useful to introduce the

notion of bulk charge Q_{bulk} which is the total charges carried by the bulk fields

$$Q_2^{bulk} = \int_{Y_2} \frac{1}{2} G_4 \wedge G_4 \ . \tag{3.1}$$

Then, the bulk charge can be understood as being related to the brane and Maxwell charges via

$$Q_2^{Maxwell} = Q_2^{brane} + Q_2^{bulk} \ . \tag{3.2}$$

To correctly quantize the supergravity solution, one should impose the discreteness condition on the Page charges, and not on Maxwell, brane, or bulk charges.

3.2 Quantization on the Stiefel cone

To illustrate the integrality of Page charges and the non-integrality of the other charges, let us first carryout the quantization procedure for the Stiefel cone.

First, consider the flux of $\tilde{F}_4$. The Stiefel geometry has vanishing fourth Betti number, so there is no G_4 to consider in M-theory, and after dimensional reduction, the IIA flux $\tilde{F}_4$ also vanishes. We are not done yet, however, because we still have to consider the Page flux (1.3), which contains a term $B_2 \wedge F_2$, and F_2 is nonvanishing in the dimensional reduction of the orbifolded Stiefel cone. Requiring the Page flux to be integer quantized imposes the quantization condition

$$\int B_2 = -\frac{l}{2k} + \frac{1}{2} \tag{3.3}$$

which we inferred independently from M-theory considerations earlier in section 2.1.

Next, we consider the quantization of flux of D2 charge through M_2. We are interested in determining the Maxwell charge when the Page charge is set to N. One finds

$$Q_2^{Maxwell} = N - \frac{l(l-2k)}{2k} \tag{3.4}$$

which can essentially be viewed as the sum of a contribution from N M2-branes and a contribution from the discrete torsion, along the lines of [14]. The Maxwell charge $Q_2^{Maxwell}$ has several notable features. First, it is not necessarily integer valued. Second, it is invariant under

$$N \to N + l, \qquad l \to l + 2k \ . \tag{3.5}$$

This is consistent with the property of Maxwell charge that it is conserved under continuous deformations corresponding to moving one of the 5-branes around the S^1 in the type IIB brane construction. Finally, $Q_2^{Maxwell}$ can go to zero or negative for some range of (N, l, k). This suggests that the entropy of the superconformal field theory is going to zero or negative, signaling a phase transition. The condition that $Q_2^{Maxwell}$ is positive is also related to the

condition necessary for supersymmetry to be unbroken as was highlighted in related contexts in [4, 5].

3.3 Quantization in the Stenzel geometry

Let us now extend our analysis of flux quantization to the case where the Stiefel cone is deformed into the Stenzel geometry, as described in Section 2.3. To keep a general set of charges under consideration, we will study the case where the Stenzel manifold has been quotiented by a Z_k orbifold action.

The most important feature of the geometry in the deep IR is its singularity structure after the orbifold has been taken. At the tip of the deformed orbifolded cone, the geometry has the local structure $(R^4 \times S^4)/Z_k$, and in particular it has two fixed points which we can think of as the north and south poles of the S^4/Z_k. At each of the fixed points, the local geometry is R^8/Z_k [6]. This geometric feature has a nice implication. The supersymmetry of the deformed Stenzel cone is consistent with adding some mobile M2-branes, and we are free to move some number of them to either of the orbifold fixed points. Then the theory on the M2-branes in the deep IR should simply be two copies of the ABJM theory.

At any finite excitation energy the theory should feel the effects of curvature and the self-dual four-form flux in the background which break the supersymmetry from $\mathcal{N} = 6$ to $\mathcal{N} = 2$. However, for issues such as charge quantization, we should be able to work in the extreme low energy limit and use our intuition from the ABJM case. In particular one might expect that it is possible to turn on discrete torsion at each singularity, and we will see that this is correct, although the torsion will be subject to some global constraints.

First we will consider the type IIA reduction of this geometry. This geometry develops a dilaton and curvature singularity near the tip. However, because the geometry asymptotes to the Stiefel cone away from the tip, and because quantization of Page fluxes in type IIA description appropriately respects Gauss law/localization of charge sources, we are able to partially constrain the discrete parameters of the supergravity ansatz. We will then continue to consider the geometry and fluxes near the core region from the M-theory perspective, and identify additional constraints which further restrict the parameters of the ansatz.

The Stenzel manifold admits the self-dual four form flux (2.14) which can be derived from a three-form potential C_3 [10]

$$C_3 = m\beta + \alpha\Omega_2 \wedge d\gamma \tag{3.6}$$

$$\beta = \frac{ac}{\epsilon^3 \cosh^4 \frac{r}{2}} \left[(2a^2 + b^2)\tilde{\sigma}_1 \wedge \tilde{\sigma}_2 \wedge \tilde{\sigma}_3 + \frac{a^2}{2}\epsilon^{ijk}\sigma_i \wedge \sigma_j \wedge \tilde{\sigma}_k \right] , \tag{3.7}$$

where Ω_2 and γ are as defined in [6].[1] Here we have added an exact term proportional to α, which does not affect the gauge invariant four-form flux. This exact term is present in the $AdS_4 \times V_{5,2}/Z_k$ system with discrete torsion [6] which is the UV limit of the Stenzel solution.

In quantizing the flux of the type IIA Page flux through the four cycle of M_2, we impose the condition

$$\int_{S^4} G_4 + nk \int_{\tilde{S}^3/Z_k} C_3 = (2\pi) n\alpha = -(2\pi l_p)^3 (l-k), \qquad n = 2 \tag{3.8}$$

which constrains α. Note that in the asymptotically conical limit, the torsion is Z_{2k}-valued, and so l takes integer values in the range $0 \le l \le 2k-1$.

In addition to this, the flux of G_4 through S^4/Z_k is independently quantized to be integral. This implies

$$\int_{S^4/Z_k} G_4 = \frac{8\pi^2}{3\sqrt{3}k} m = (2\pi l_p)^3 q \tag{3.9}$$

for integer q. This constraint has no counterpart in the Stiefel cone as neither the S^4 cycle nor the self-dual 4-form exist in that case.

Now let us consider the quantization conditions that arise from considering M-theory near the orbifold fixed points; we will show that the expected charges at the singularities are compatible with the IIA calculations. At the north pole of S^4/Z_k, the pull-back of G_4 on the R^4/Z_k fiber was computed in [6]:

$$\frac{1}{(2\pi l_p)^3} \int_{R^4/Z_k} G_4 = \frac{q}{2} \equiv \frac{\tilde{M}}{2} = M \tag{3.10}$$

where M and $\tilde{M}$ are the variables used in [6]. This means that the integral of C_3 (including both the nontrivial flux and the discrete torsion contribution) on S^3/Z_k at the north pole is

$$\frac{1}{(2\pi l_p)^3} \int_{\tilde{S}^3/Z_k} C_3 = -\frac{l}{2k} + \frac{1}{2} - \frac{q}{2} \; . \tag{3.11}$$

Suppose that at the north pole we impose the condition that the system is described by charges as in the ABJ case with l^N units of discrete torsion (including a shift by 1/2 a unit as discussed in [4].) This is compatible with (3.11) provided that

$$-\frac{l^N}{k} + \frac{1}{2} = -\frac{l}{2k} + \frac{1}{2} - \frac{q}{2} \tag{3.12}$$

or equivalently

$$l^N = \frac{l + kq}{2}. \tag{3.13}$$

[1] For the interested reader, γ is the angular coordinate which is quotiented by the orbifold action, and Ω_2 is proportional to the geometric flux associated with γ.

At the south pole, the computation is very similar, except that the flux quantum q appears with a minus sign:

$$l^S = \frac{l - kq}{2} \tag{3.14}$$

The difference in the pull-back of C_3 between the north and the south pole is just the total flux q, while the discrete torsion contribution must be the same at the north and south poles because the torsion has no associated flux.[2]

How should we interpret the formulas (3.13) and (3.14)? The first thing to note is that l^N and l^S are equal mod k, so if they had described decoupled systems we would have said that they were equivalent up to a large gauge transformation. However, they are not decoupled, and there is no large gauge transformation that sets them equal to each other. Instead, the picture that has emerged is that l^N and l^S locally appear to describe the same torsion, but globally there is a topologically nontrivial twist relating them, and the winding number of the twist is just the number of units of G_4 flux in S^4/Z_k.

The second thing to note is that in the local ABJ models at the north and south poles, l^N and l^S should themselves be integers, or in other words $l - kq$ must be even. This means that for a given q and k, l must take either only even or only odd values. In the undeformed theory, l described a Z_{2k}-valued discrete torsion, but we see that our local considerations at the tip of the Stenzel geometry remove half of the possible values of l, and the discrete torsion in the deformed case is Z_k-valued. This phenomenon is reminiscent of the deformed conifold; the "singular" conifold admits a Z_2-valued discrete torsion which is not present in the deformed conifold [16].

We can now examine the quantization of the six form flux though M_2 in IIA or the 7-form flux through $V_{5,2}$ in M-theory which measures the charge of D2/M2 branes in this background.

One way to approach this issue is to first examine the brane charges present in this setup. Before adding any explicit 2-branes, there are 2-brane charges arising from the discrete torsion at the Z_k fixed points at north and south poles [14]. These should have the same form as what was computed in [4], so we find

$$Q_2^{torsion} = \left(-\frac{l^N}{k} + \frac{1}{2}\right) + \left(-\frac{l^S}{k} + \frac{1}{2}\right) = -\frac{l(l-2k)}{4k} - \frac{kq^2}{4}\ . \tag{3.15}$$

If, in addition, we were to introduce N 2-branes which can be at any point in the Stenzel

[2]In the coordinates of [10,15], the $U(1)_b$ quotient as defined in [6] is imposed on the angular coordinate ϕ_2. With this choice of $U(1)$ action, the poles of the S^4/Z_k are located at $(\tau = 0, \alpha = \pi/2, \psi = 0, \pi)$. In the vicinity of the poles, one can check that the one-forms $\tilde{\sigma}_i$ differ by a sign, $\tilde{\sigma}_i(N) = -\tilde{\sigma}_i(S)$, so the three-form β in (3.7) also changes by a sign from the north pole to the south pole.

geometry, there will be an additional contribution of N to the brane charge

$$Q_2^{brane} = N - \frac{l(l-2k)}{4k} - \frac{kq^2}{4} \ . \tag{3.16}$$

Since Maxwell charge is the sum of brane charge, and since the bulk charge is given by

$$Q_2^{bulk} = \frac{1}{(2\pi l_p)^6} \int_{\mathcal{M}_8} \frac{1}{2} G \wedge G = \frac{2^{11} m^2 \mathrm{vol}(V_{5,2})}{(2\pi l_p)^6 3^6} = \frac{kq^2}{4} \tag{3.17}$$

we infer that

$$Q_2^{Maxwell} = N - \frac{l(l-2k)}{4k} \ . \tag{3.18}$$

It also follows that the Page charge $Q_2^{Page} = N$.

This result is gratifying for several reasons. First, this result reflects the accounting of all identifiable charge sources in an otherwise consistent and smooth M-theory background aside from the orbifold fixed point. The final answer is the same as what we inferred for the undeformed Stiefel cone (3.4). It then follows that the gauge invariant Maxwell charge is invariant under the shifts

$$N \to N + l, \qquad l \to l + 2k \tag{3.19}$$

which arises naturally from several perspectives mentioned earlier.

The only additional constraint imposed by the Stenzel deformation is the restriction on the parity of l so that l is congruent to kq mod 2. This is far milder than what was found in [6].

4 Discussion

In this article, we reviewed the quantization of fluxes in warped Stiefel cone and its Stenzel deformation which is conjectured to be the holographic dual of $\mathcal{N} = 2$ Chern-Simons matter theory in 2+1 dimensions. We described the subtle difference between several different yet related notions of charges, and recovered a structure compatible with the pattern of Hanany-Witten brane creation effects and duality cascades.

There are a number of interesting features which one can infer from the structure of the gravity solution. Q_2^{brane} is a measure of the number of degrees of freedom in the deep infrared of this system. When Q_2^{brane} is zero or negative, we expect the system to break supersymmetry and flow to a different universality class of vacuum as was the case for many related system [4, 5]. It would be very interesting to better understand the nature of the effective low energy physics when the system is in this new phase. This question can be addressed in the simple context of $k = 1$ where there are no Z_k orbifold fixed points, and by

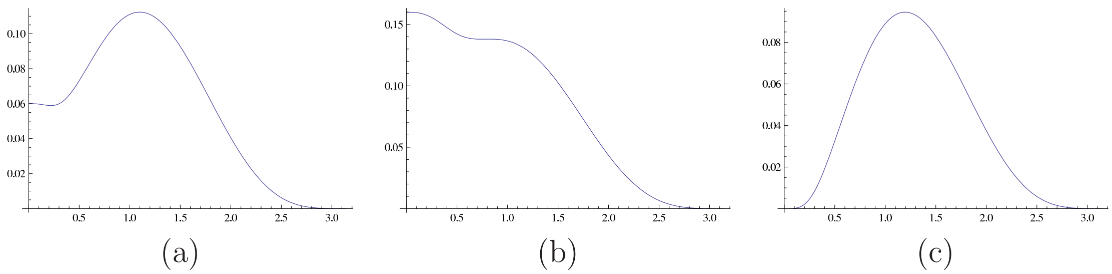

Figure 1: Potential $V(\psi)$ for p anti D3-brane blowing up to an NS5-brane wrapping an S^2 of fixed latitude in ψ in S^3 at the tip of the Klebanov-Strassler solution. (a), (b), and (c) corresponds to $p/M = 0.03$, $p/M = 0.08$, and $p/M = -0.03$, respectively. These figures originally appeared in figure 2 of [17].

taking q to be even, we can even set $l = 0$ and disregard the contribution from the discrete torsion.

One way to probe the fate of pushing the system which is slightly perturbed into this new phase is to start with a background with q large but $Q_2^{brane} = 0$ (which can easily be arranged for q even and $k = 1$). Consider now adding $p \ll q$ anti M2-brane as a probe. This setup is very similar to adding anti D3-brane in warped deformed conifolds [17] which has received a lot of attention (and controversy) as a possible prototype as a gravity dual of a metastable vacua [18–20]. For the Stenzel manifold, the effective action of the brane probe undergoing a KPV-like transition [17] works essentially in the same way as is illustrated in figure 1. However, from the point of view of the bound $Q_2^{brane} > 0$, one expects the stable supersymmetric minima not to exist when p anti M2-branes are introduced.

Tentatively, we interpret these facts as follows. The computation of the potential $V(\psi)$ neglected the backreaction of the anti-branes, and when the number of anti-branes is parametrically small ($p \ll q$) this probe approximation is valid. In particular, the existence of the non-BPS local minimum in 1.(a) is a robust prediction in this limit. However, when the state in the metastable false vacuum illustrated in figure 1.(a) tunnels to the putative "true" vacuum, the amount of charge carried by the probe grows to $q - p$ which is not parametrically small compared to q. The backreaction due to this charge can be significant, and so the computation of the tunneling potential is (at least) not obviously self-consistent. It is tempting to speculate that the supersymmetric vacuum might actually be spurious and that the non-BPS local minimum is the global minimum which characterizes the dynamics in the $Q_2^{brane} < 0$ phase of these theories up to corrections suppressed by p/q.

Similar considerations apply to the BPS domain wall one constructs for $p < 0$ for which the KPV potential has the form illustrated in figure 1.(c). This domain wall can also be viewed as arising from wrapping an M5-brane on S^4 at the tip of the Stenzel manifold. A

5-brane wrapped on a 4-cycle is effectively a string, and in 2+1 dimensions, a string forms a domain wall. It would be very interesting to understand the nature of vacua separated by these domain walls. Since M5-brane wrapped on S^4 with q units of flux must have q additional M2-branes ending on it to cancel the anomaly, some quantum numbers of the vacuum must shift to reflect this. Nonetheless, one expects the Maxwell charges $Q_2^{Maxwell}$ and $Q_4^{Maxwell}$ to be invariant as one crosses the domain wall, as these charges are conserved. Making complete sense of these expectations requires taking the full back reaction of the M5-brane and the q anomalous M2-branes into account. Unfortunately, q M2-branes can not be treated reliably as a probe, making systematic analysis of these issues a challenge.

Let us also mention that similar issues of stable/metastable non-BPS vacua, domain wall, and low energy effective field theories can be discussed in the closely related B_8 system building on the analysis of [5] and [21]. Quantization of charges and the enumeration of brane, Maxwell, and Page charges for this system was carried out in [5]. Here, however, we encounter one additional puzzle. It was argued in [21] that the 4-form flux through S^4 at the tip of the B_8 cone is half integral as a result of the shift originally due to Witten [22]. This would appear to require half integer units of M2-branes to end on the domain wall made by wrapping the M5 on the S^4. Of course, the number of M2's ending on an M5 is constrained to be an integer. Perhaps this is indicating that odd number of M5-branes are forbidden from wrapping the S^4. Alternatively, this paradox is another manifestation of not systematically taking the back reaction of the domain wall into account.

Finally, let us emphasize that for the time being, the concrete field theory interpretation of the Stenzel deformation and the quantum number q is not known. The gravity dual suggests that the parameter q is important for both the IR and the UV physics. At large radius, q is related to the total number of units of M2-brane charge generated by the cascade, which in turn affects the UV gauge symmetry. Near the tip of the Stenzel geometry, the G_4 flux is nonvanishing so q should also appear in the data of the IR field theory. Of course, q can only be nonzero when the geometry is deformed. Martelli and Sparks conjectured that this deformation was related to turning on a particular mass term on the field theory side. One can indeed see that the null geodesic can travel from boundary at infinity to the core in finite field theory time, and so the spectrum of glueball-like states will exhibit a discrete structre whose scale is set by the deformation. If this conjecture is correct, it would suggest that the field theory confines because of a mass deformation (reminiscent of the $\mathcal{N} = 1^*$ theory in $d = 4$ [23] and the mass deformed ABJM theory [24–27]) rather than as a dynamical effect, as is the case in the Klebanov-Strassler system [28]. It should be very interesting to understand this theory better.

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A Charge quantization and duality transformations

One of the subtle features arising in quantizing the supergravity background in this article is the fact that some fluxes lifts or dualizes to a quantity encoded not by the period of a field strength, but by a quantity like the discrete torsion which is the period of a potential field. In this appendix, we illustrate several examples, in a simpler context, giving rise to similar subtleties.

A.1 Charge quantization for duals of $TN \times S^1$

The KK-monopole, also known as the Taub-NUT space, is a well known Ricci-flat gravitational background. The metric for $TN \times S^1$ has a simple form

$$\left(1+\frac{R_1}{2r}\right)(dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2))+\frac{R_1^2}{\left(1+\frac{R_1}{2r}\right)}\left(d\psi+\frac{1}{2}\cos\theta d\phi\right)^2+R_2^2 d\eta^2 \ . \qquad \text{(A.1)}$$

We take ϕ, ψ, and η to have period 2π, and $0 \leq \theta \leq \pi$. R_1 and R_2 are the radius of S^1 parametrized by ψ and η, respectively.

Being Ricci flat, this metric can easily be embedded in M-theory. Reducing to IIA along η will give rise to a KK5-brane in type IIA string theory. Reducing to IIA along ψ will give rise to a D6 brane extended along η.

Consider a general linear transformation on the coordinates ψ and η

$$\eta = a\eta' + b\psi', \qquad \psi = c\eta' + d\psi' \ . \qquad \text{(A.2)}$$

This will modify the last two terms of (A.1) to

$$\frac{R_1^2 R_2^2}{(c^2R_1^2+a^2R_2^2V)}\left((ad-bc)d\psi'-\frac{a}{2}\cos\theta d\phi\right)^2+\left(a^2R_2^2+\frac{c^2R_1^2}{V}\right)(d\eta'+A_1)^2 \qquad \text{(A.3)}$$

with

$$A_1 = \left(\frac{cR_1^2}{a(c^2R_1^2 + a^2R_2^2 V)} + \frac{1}{(ad-bc)}\frac{b}{a} \right) \left((ad-bc)d\psi' + \frac{a}{2}\cos\theta d\phi \right) - \frac{1}{(ad-bc)}\frac{b}{2}\cos\theta d\phi \ . \tag{A.4}$$

At the level of classical supergravity, this is a solution generating transformation, but not all of the solutions obtained in this fashion are sensible backgrounds of string theory. Rather, there is a certain discrete subset of these solutions which is consistent with charge quantization.

A.2 Twisted Z_p orbifold

One example of such a discrete subset is to take

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & q/p \\ 0 & 1/p \end{pmatrix} \tag{A.5}$$

and impose the periodicity $\eta' = \eta' + 2\pi$ and $\psi' = \psi' + 2\pi$. This can be viewed as a twisted Z_p orbifold of $TN \times S^1$ as outlined in (3.4) of [29]. When reducing to IIA, one finds a RR 1-form potential

$$A_1 = \left(\frac{q}{p}d\psi' + \frac{q}{2}\cos\theta d\phi \right) - \frac{q}{2}\cos\theta d\phi = \frac{q}{p}d\psi' \ . \tag{A.6}$$

Upon further T-dualizing this background along the ψ' direction, we obtain a supergravity solution for a (p, q) 5-brane[3] smeared along the ψ' direction. This can easily be seen from the RR 2-form potential

$$A_2 = -\frac{q}{2}\cos\theta d\phi \wedge d\tilde{\psi}' \tag{A.7}$$

and the NSNS 2-form potential

$$B_2 = -\frac{p}{2}\cos\theta d\phi \wedge d\tilde{\psi}' \tag{A.8}$$

which one finds from the duality transformation. The 3-form field strength is closed and naturally encodes the flux through $S^2 \times S^1$ parametrized by θ, ϕ, and $\tilde{\psi}'$ for arbitrary r.

The issue stems from attempting to understand the q units of D6 charge from the IIA description *prior* to taking the T-duality along the ψ' direction. One expects the D6 charge to be encoded by the flux of the RR 2 form field strength

$$F_2 = dA_1 \tag{A.9}$$

[3]The p and q are switched from what is in [29] so that p counts the number of NS5-brane and q counts the number of D5-brane in the dual IIB description.

but this *vanishes* for the background (A.6) under consideration. For this example, the hint for where the quantum number of the D5 charge is encoded in the IIA description is staring at our face in equation (A.6). It is the fractionally valued vector potential arising as a result of the non-trivial twist, q, in the Z_p. This may be thought of as the simplest example illustrating the point that charge is sometimes encoded in the period of a potential, i.e. a Wilson line, rather than the field strength.

A.3 $SL(2, Z)$ dual of $TN \times S^1$

Let us now consider a different example, where we take the $SL(2, Z)$ subset of the general linear transformation (A.2). In this case, we have $ad - bc = 1$, simplifying (A.4) to

$$A_1 = \left(\frac{cR_1^2}{a(c^2R_1^2 + a^2R_2^2V)} + \frac{b}{a} \right) \left(d\psi' + \frac{a}{2} \cos\theta d\phi \right) - \frac{b}{2} \cos\theta d\phi \ . \tag{A.10}$$

Once again, T-dualizing on ψ' will give rise to a RR 2-form

$$A_2 = -\frac{b}{2} \cos\theta d\phi \wedge d\tilde{\psi}' \ . \tag{A.11}$$

In fact, if we take $a = p$ and $b = q$, the IIB 5-brane charges are identical to the example in the previous section although the background differs in the asymptotic value of the axiodilaton.

The puzzle, once again, is the status of the D6 charge in the IIA frame. This time, the RR 2-form field strength does not vanish, so one might try to define a D6-brane charge by integrating F_2 over a suitable 2-cycle. However, no such nonsingular 2-cycle exists. For example, integrating on the natural S^2 parametrized by θ, ϕ would give a charge that depends on the radius r. This apparent failure of the Gauss law can be traced to this S^2 not actually being a well-defined 2-cycle.

Notice that in the ordinary IIA reduction of (A.1) on the circle parametrized by ψ, the procedure of integrating F_2 on the S^2 at fixed radius is the correct one for counting the number of D6 branes. On the other hand, for the IIA reduction on η (or in M-theory) the integrality of the D6 brane charge follows from demanding that the KK5 metric does not have a singularity. In a generic duality frame, such as the one given by reduction on ψ', neither condition is correct.

Instead, one might try to define a modified flux quantization condition that mixes the flux and geometry in such a way as to obtain a conserved charge. This can be done by considering the combination

$$Q_{D6} = \frac{1}{2\pi} \left(\int_{S^2} F_2 + \frac{a}{ad - bc} \int_{S^1} A_1 \right) = \frac{b}{ad - bc} \tag{A.12}$$

where S^2 is the 2-sphere parametrized by (θ, ϕ), and S^1 is the circle parametrized by ψ'. Note that this reduces to the same prescription as in the previous subsection if F_2 happens to vanish. In fact, one could have also considered applying the prescription of reading off the period of A_1 precisely at the radius $r = 0$ where F_2 would have vanished.

Formally, this procedure is equivalent to considering

$$Q_6 = \frac{1}{2\pi} \int_{S^2} (F_2 + f \wedge A_1) \tag{A.13}$$

discussed briefly in appendix A of [4] based on the language of [30–33]. Although the procedure of computing the period of this "modified" flux will give the correct answer, it is somewhat unsettling that the procedure is not generally covariant. The aim of this article is to identify the origin of this charge from discrete data of the potential.

A.4 Quantization of gravity duals of field theories in 2+1 dimensions

Let us now examine the quantization of fluxes in gravity duals of various 2+1 dimensional field theories and compare their features with the example of the previous section.

We will begin by reviewing the case of ABJM and ABJ for which the gravity theory is M-theory on $AdS_4 \times S^7/Z_k$ [1,2]. The transverse S^7/Z_k has a torsion 3-cycle on which one can wrap l M5-branes in the range $0 \leq l < k$. These M5-branes will not source M-theory 4-form flux. Instead, they give rise to discrete torsion parametrized by a flat C_3 in the background, supported on the S^3/Z_k. This is quite similar to the case of the one form potential (A.6) in the case of twisted Z_p orbifold of $TN \times S^1$ we described in section A.2 and A.3. There are two refinements to this story.

One is that the quantization condition for the discrete torsion has an anomalous shift due to the Freed-Witten anomaly, and reads

$$k \int_{S^3/Z_k} C_3 = l - \frac{k}{2} \ . \tag{A.14}$$

The presence of Freed-Witten anomaly was inferred in the IIA description of this background in [4]. At the moment, it is not clear how one understands this shift strictly in the M-theory perspective, but since it is required in the IIA reduction, we will adopt it in the M-theory lift as well. One can simply view this as an overall shift in the charge lattice. As we will see below, this shift turns out to be consistent with a rather non-trivial consistency test.

The second refinement concerns the relation between the M2 charge and the radius of anti de Sitter geometry. In the absence of discrete torsion, the radius of anti de Sitter space is directly proportional to the number of M2 branes giving rise to the near horizon

$AdS_4 \times S^7$ geometry. In the presence of discrete torsion, however, the relation receives a correction. This issue was investigated originally in [14] which left out the contribution from the Freed-Witten anomaly. Taking the Freed-Witten anomaly into account [4], one finds that

$$R^4 = (2^5 \pi l_p^4) \frac{Q_2}{k} \tag{A.15}$$

where

$$Q_2 = N - \frac{l(l-k)}{2k} + Q_{curv} \tag{A.16}$$

with

$$Q_{curv} = -\frac{1}{24}\left(k - \frac{1}{k}\right) \tag{A.17}$$

is the contribution from the $C_3 \wedge R^4$ correction to the M-theory action.

In the IIA reduction along the Hopf fiber of S^3/Z_k, the Q_2 can also be written

$$Q_2 = \left(N + \frac{k}{8}\right) + \left(l - \frac{k}{2}\right) b + \frac{1}{2} k b^2 + Q_{curv}, \qquad b = -\frac{l}{k} + \frac{1}{2} \tag{A.18}$$

where b is the pull-back of B_2 on S^2 level surface of R^4/Z_k reduced on S^1. This expression makes the interpretation of Q_2 as including the contribution from the B-field in the Wess-Zumino term for k D6-brane and $l - k/2$ D4-branes, including the Freed-Witten shift, manifest. There is also a fractional shift in the D2 charge by $k/8$ and from Q_{curv}. The $k/8$ shift can be viewed as following from the Freed-Witten anomaly.

Recently, in a very impressive paper [34], the planar free energy of the ABJM/ABJ field theory was computed in the strong coupling limit on the field theory side using localization and matrix model techniques. They found that up to numerical constants (which they also compute), the free energy is proportional to $Q_2^{3/2}$. This is a rather non-trivial test for the consistency of the details of the shifts in curvature due to discrete torsion, including the detailed form of the effects of Freed-Witten anomaly. They in fact confirm specifically the presence of a shift in the D2 charge by $k/8 - k/24 = k/12$ units.[4]

Let us also comment in passing that the positivity of the anti de-Sitter radius (excluding the contribution from the curvature) is equivalent to the condition for preserving supersymmetry implied by generalized s-rule [4, 5].

Finally, let us discuss the generalization of (A.14) when the background supports non-trivial G_4, as is the case for the Stenzel geometry. In this case, the pull-back of C_3 can vary in such a way that can be cancelled by the pull-back of G_4. By subtracting this contribution, one arrives at (3.8) which can be interpreted as the M-theory lift of the Page charge (1.3).

[4]The analysis of [34] was in the planar limit $k \gg 1$ where N/k and l/k were kept fixed. It is possible that Q_2 has corrections suppressed by $1/k$ that has not yet been accounted for in the gravity side.

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